Q.4 long

★ Electric Potential Due to a Dipole

The potential at point P due to the charge +q and −q is given by

$$\phi(\vec{r}) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{|\vec{r}-\vec{r}'-\vec{l}|} - \frac{1}{|\vec{r}-\vec{r}'|}\right] \quad \text{——— (4)}$$

Now;

$$\frac{1}{|\vec{r}-\vec{r}'-\vec{l}|} = |\vec{r}-\vec{r}'-\vec{l}|^{-1}$$

$$= \left[(\vec{r}-\vec{r}'-\vec{l})^2\right]^{-1/2}$$

$$= \left[|\vec{r}-\vec{r}'|^2 - 2(\vec{r}-\vec{r}')\vec{l} + l^2\right]^{-1/2}$$

$$= |\vec{r}-\vec{r}'|^{-1}\left[1 - \frac{2(\vec{r}-\vec{r}')\cdot\vec{l}}{|\vec{r}-\vec{r}'|^2} + \frac{l^2}{|\vec{r}-\vec{r}'|^2}\right]^{-1/2}$$

As $l <<< |\vec{r}-\vec{r}'|$

$$\therefore \frac{1}{|\vec{r}-\vec{r}'-\vec{l}|} = \frac{1}{|\vec{r}-\vec{r}'|}\left[1 - \frac{2(\vec{r}-\vec{r}')\cdot\vec{l}}{|\vec{r}-\vec{r}'|^2}\right]^{-1/2}$$

By binomial theorem,

$$\frac{1}{|\vec{r}-\vec{r}'-\vec{l}|} = \frac{1}{|\vec{r}-\vec{r}'|}\left[1 + \frac{|\vec{r}-\vec{r}'|\cdot\vec{l}}{|\vec{r}-\vec{r}'|^2}\right]$$

Neglect the higher terms

Put this value in eq^n (4) we get,

$$\phi(\vec{r}) = \frac{q}{4\pi\epsilon_0}\left[\cancel{\frac{1}{|\vec{r}-\vec{r}'|}} + \frac{(\vec{r}-\vec{r}')\cdot\vec{l}}{|\vec{r}-\vec{r}'|^3} - \cancel{\frac{1}{|\vec{r}-\vec{r}'|}}\right]$$

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$$\phi(\vec{r}) = \frac{q}{4\pi\epsilon_0} \frac{(\vec{r}-\vec{r'})\cdot\vec{l}}{|\vec{r}-\vec{r'}|^3}$$

In terms of dipole moment, it may be written as

$$\phi(\vec{r}) = \frac{\vec{p}\cdot(\vec{r}-\vec{r'})}{4\pi\epsilon_0 |\vec{r}-\vec{r'}|^3} \quad \text{——— (5)}$$

where $\vec{p} = q\vec{l}$

This is the general expression for the electric potential produced by an electric dipole.

If $\vec{r'}=0$ then from eq^n (5) we may be write;

$$\boxed{\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{\vec{r}\cdot\vec{p}}{r^3}}$$

OR

$$\boxed{\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{p\cos\theta}{r^2}}$$

where θ is the $\angle$le b/w $\vec{r}$ and $\vec{p}$. This expression may be used to write radial and transverse components E_r and E_θ of the electric field $\vec{E}$ at the point $P(r,\theta)$

$$E_r = -\frac{\partial\phi}{\partial r}$$

$$= -\frac{\partial}{\partial r}\left[\frac{1}{4\pi\epsilon_0} \frac{p\cos\theta}{r^2}\right]$$

$$\boxed{E_r = \frac{1}{4\pi\epsilon_0}\frac{2p\cos\theta}{r^3}}$$

$$E_\theta = \frac{-1}{r}\frac{\partial\phi}{\partial\theta}$$

$$= \frac{-1}{r}\frac{\partial}{\partial\theta}\left[\frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}\right]$$

$$\boxed{E_\theta = \frac{1}{4\pi\epsilon_0}\frac{p\sin\theta}{r^3}}$$

Then, the magnitude of the resultant field is

$$E = \sqrt{E_r^2 + E_\theta^2}$$

$$= \sqrt{\left[\frac{1}{4\pi\epsilon_0}\frac{2p\cos\theta}{r^3}\right]^2 + \left[\frac{1}{4\pi\epsilon_0}\frac{p\sin\theta}{r^3}\right]^2}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{p}{r^3}\sqrt{1+3\cos^2\theta}$$

If α is the angle b/w $\vec{E}$ and $\vec{r}$ then

$$\tan\alpha = \frac{E_\theta}{E_r}$$

$$\tan\alpha = \frac{\frac{1}{4\pi\epsilon_0}\frac{p\sin\theta}{r^3}}{\frac{1}{4\pi\epsilon_0}\frac{2p\cos\theta}{r^3}}$$

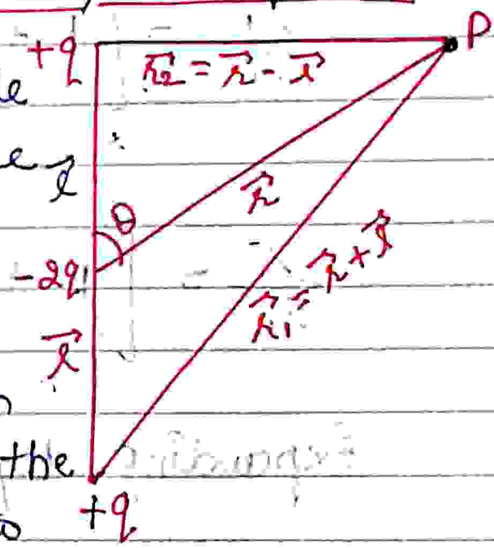
$$\tan\alpha = \frac{1}{2}\tan\theta$$

$$\boxed{\alpha = \tan^{-1}\left(\frac{1}{2}\tan\theta\right)}$$

★ Electric Quadrupole

An electric quadrupole is that which consists of two equal and opposite dipoles that do not coincide in space so that their electric effects do not quite cancel each other at distant points.

• Electric potential due to a Quadrupole



Let us consider ~~eg~~ a linear quadrupole with charge $-2q$ in the middle and charges $+q$, $+q$ at the ends.

Let P be distant point with position vector $\vec{r}$ at which the electrostatic potential is to find out.

The separation $\vec{l}$ b/w the charge is very small compared to the distance $\vec{r}$.

Let $\vec{r_1}$ and $\vec{r_2}$ be the distances of the point P from the charges $+q$, $+q$. Thus the potential at P due to the quadrupole is

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r_1} + \frac{q}{r_2} - \frac{2q}{r}\right]$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{r}\left[\frac{r}{r_1} + \frac{r}{r_2} - 2\right] \quad \text{--- (7)}$$

$$\vec{r_1} = |\vec{r} + \vec{l}|$$

$$= \left[(\vec{r}+\vec{l})\cdot(\vec{r}+\vec{l})\right]^{1/2}$$

$$= \left[r^2 + l^2 + 2\vec{r}\cdot\vec{l}\right]^{1/2}$$

$$= r\left[1 + \frac{l^2}{r^2} + \frac{2\vec{r}\cdot\vec{l}}{r^2}\right]^{1/2}$$

$$\frac{r_1}{r} = \left[1 + \left\{\frac{l^2}{r^2} + \frac{2\vec{r}\cdot\vec{l}}{r^2}\right\}\right]^{1/2}$$

'OR'

$$\frac{r}{r_1} = \left[1 + \left\{\frac{l^2}{r^2} + \frac{2\vec{r}\cdot\vec{l}}{r^2}\right\}\right]^{-1/2}$$

Expanding by binomial theorem, we get,

$$\frac{r}{r_1} = 1 - \frac{1}{2}\left(\frac{l^2}{r^2} + \frac{2\vec{r}\cdot\vec{l}}{r^2}\right) + \frac{(-1/2)(-1/2-1)}{\underline{|2}}\left[\frac{l^2}{r^2} + \frac{2\vec{r}\cdot\vec{l}}{r^2}\right]^2$$

$$\because \vec{r}\cdot\vec{l} = rl\cos\theta$$

$$\frac{r}{r_1} = 1 - \frac{1}{2}\left[\frac{l^2}{r^2} + \frac{2l\cos\theta}{r}\right] + \frac{3}{8}\left[\frac{l^2}{r^2} + \frac{2l\cos\theta}{r}\right]^2 - \cdots$$

As $l <<< r$, neglecting the higher terms Then we get

$$\frac{r}{r_1} = 1 - \frac{1}{2}\frac{l^2}{r^2} - \frac{l}{r}\cos\theta + \frac{3}{8} \times \frac{4l^2\cos^2\theta}{r^2}$$

$$\frac{r}{r_1} = 1 - \frac{l}{r}\cos\theta + \frac{l^2}{2r^2}(3\cos^2\theta - 1)$$

Similarly;

$$\frac{r}{r_2} = 1 + \frac{l}{r}\cos\theta + \frac{l^2}{2r^2}(3\cos^2\theta - 1)$$

$$\therefore \frac{r}{r_1} + \frac{r}{r_2} = 2 + \frac{l^2}{r^2}(3\cos^2\theta - 1)$$

Put this value of $\frac{r}{r_1} + \frac{r}{r_2}$ in eqn. (1) we get;

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r}\left[\cancel{2} + \frac{l^2}{r^2}(3\cos^2\theta - 1) - \cancel{2}\right]$$

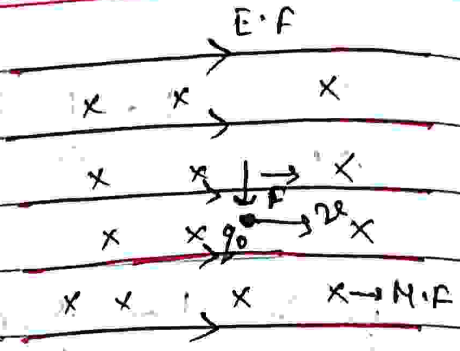
$$\boxed{\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} q \frac{l^2}{r^3}(3\cos^2\theta - 1)}$$

This is the required expression.

★ Lorentz Force

The Lorentz force may be defined as "the force experienced by charged particle moving in a space where both electric and magnetic field exist. is known as Lorentz force."

Let us consider a test charge q_0 is moving in a space where both electric field $\vec{E}$ and magnetic field $\vec{B}$ are present. Then the force on a charge will be



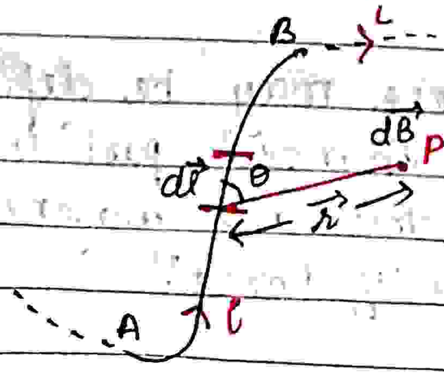
$$\boxed{\vec{F} = q_0\left[\vec{E} + \vec{v} \times \vec{B}\right]}$$

This force is called <u>Lorentz force</u> and this eq. is called <u>Lorentz force equation.</u>

If $\vec{E} = 0$ then
$$\vec{F} = q_0(\vec{v} \times \vec{B})$$
'OR'
$$\boxed{F = q_0 vB\sin\theta}$$

★ Biot - Savart's Law

A current carrying conductor produces a magnetic field around it. The magnitude and direction of this field at a point can be expressed by means of a law determined experimentally by Biot and Savart's and called as Biot-Savart's law.



Let 'i' be the current flow in the current carrying conductor AB. and 'P' be a point at which the field $(\vec{B})$ is to be find out. Let $d\vec{l}$ be the length of one such element. Let $\vec{r}$ be a position vector from the element to the point P. Then, according to Biot - Savart's law, the magnetic field induction $(d\vec{B})$ at point P due to the current element $d\vec{l}$ is given by

$$d\vec{B} \propto \frac{i\, d\vec{l} \times \vec{r}}{r^3}$$

'OR'

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\, d\vec{l} \times \vec{r}}{r^3}$$

'OR'

$$\boxed{dB = \frac{\mu_0}{4\pi} \frac{i\, dl \sin\theta}{r^2}} \quad \text{——— (1)}$$

Where $\frac{\mu_0}{4\pi}$ is a proportionality constant and

μ_0 is called permeability constant. Its value in S.I system is $4\pi \times 10^{-7}$ Weber/Ampere meter.

This equation represents the Biot - Savart's law. Therefore, the resultant field induction due to the whole current carrying conductor at point P is given by

$$\boxed{B = \int dB}$$

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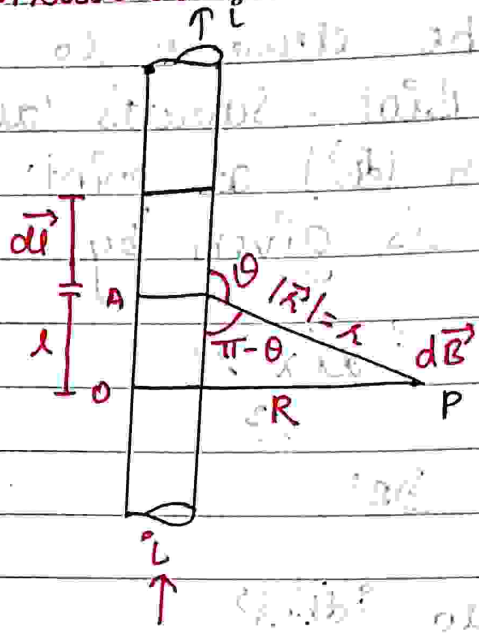
- ## Applications of Biot-Savart's law

1. Magnetic field due to a long, straight current carrying conductor:



Let us consider an infinitely long conductor which is placed in a vacuum and carrying a current 'i' amp.

Let P be a point at which the magnetic field $\vec{dB}$ is to be find out.

Here OP = R and OA = l,
Let $\vec{dl}$ is the current element at A. Then acc. to Biot-Savart's law the magnetic field induction $\vec{dB}$ at P due to current element $\vec{dl}$ is

$$\vec{dB} = \frac{\mu_0}{4\pi} \, i \, \frac{\vec{dl} \times \vec{r}}{r^3}$$

Its magnitude is given by

$$\boxed{dB = \frac{\mu_0}{4\pi} \frac{i \, dl \sin\theta}{r^2}} \longrightarrow (1)$$

In ΔOAP

$$\sin(\pi - \theta) = \frac{R}{r}$$

$$\sin\theta = \frac{R}{r}$$

$$r = \frac{R}{\sin\theta}$$

$$r = R\operatorname{cosec}\theta$$

Similarly;

$$\tan(\pi - \theta) = \frac{R}{l}$$

$$-\tan\theta = \frac{R}{l}$$

$$l = -R\cot\theta$$

Diff. above eqn. w.r.t 'θ' we get

$$\frac{dl}{d\theta} = R\operatorname{cosec}^2\theta$$

$$dl = R\operatorname{cosec}^2\theta \cdot d\theta$$

Putting the value of r and dl in eqn. (1), we get

$$dB = \frac{\mu_0}{4\pi} \frac{iR\operatorname{cosec}^2\theta d\theta \cdot \sin\theta}{R^2 \operatorname{cosec}^2\theta}$$

$$dB = \frac{\mu_0}{4\pi} \frac{i\sin\theta \, d\theta}{R}$$

Therefore the magnitude of the magnetic field at P due to the whole conductor is

$$B = \int dB$$

$$B = \int \frac{\mu_0}{4\pi} \frac{i\sin\theta}{R} d\theta$$

$$= \frac{\mu_0}{4\pi} \frac{i}{R} [-\cos\theta]_0^{\pi}$$

$$= \frac{\mu_0}{4\pi} \frac{i}{R} [-\cos\pi + \cos 0]$$

$$= \frac{\mu_0}{4\pi} \frac{i}{R} [-(-1)+1]$$

$$\boxed{B = \frac{\mu_0 i}{2\pi R}}$$

This is the expression for the magnetic field intensity due to a long straight conductor.

2. Magnetic field along the axis of a circular current loop

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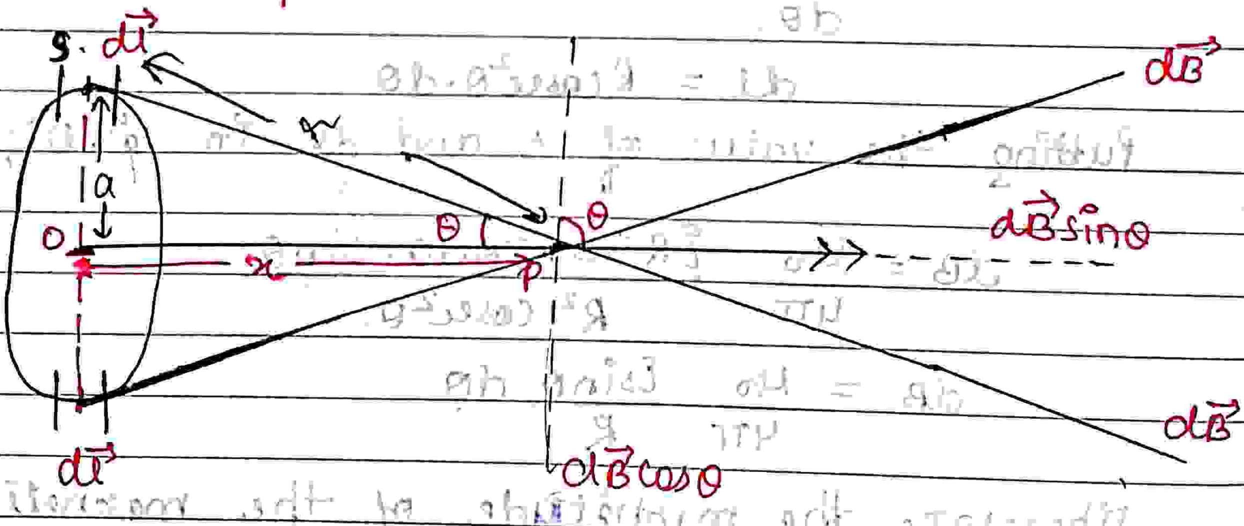


Let there be a circular coil of radius 'a' carrying a current 'i'. Let P be a point on the axis of the coil, distance 'x' from the centre at which the field is to be find out.

Let $\vec{dl}$ be a current-element at the top of the coil. If $\vec{r}$ be the displacement vector from the current-element to 'P'. Then from Biot-Savart's law, the magnetic field induction at P due to the current-element $\vec{dl}$ is given by

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\,\vec{dl} \times \vec{r}}{r^3}$$

OR,

$$dB = \frac{\mu_0}{4\pi} \frac{i\,dl \sin\theta}{r^2}$$

Because the ∠le b/w $\vec{dl}$ and $\vec{r}$ is 90°. Then

$$dB = \frac{\mu_0}{4\pi} \frac{i\,dl}{r^2} \quad \text{———} \quad (1)$$

$\vec{dl}$ and $\vec{r}$ both are ⊥r to the direction of $d\vec{B}$. The vertical components of the magnetic field $dB\cos\theta$ is cancelled due to the equal and opposite direction. But the horizontal components are effective.

Thus the resultant field $\vec{B}$ at P due to the complete loop is given by

$$B = \int dB \sin\theta$$

Put the value of dB in above eqn. from eqn. (1) we get;

$$B = \int \frac{\mu_0}{4\pi} \frac{i\,dl \sin\theta}{r^2}$$

$$B = \frac{\mu_0}{4\pi} \frac{i}{r^2} \sin\theta \int dl$$

$$\because \int dl = 2\pi a$$

$$\therefore B = \frac{\mu_0}{\cancel{4\pi}_{2}} \frac{i}{r^2} \sin\theta \times 2\cancel{\pi} a$$

$$B = \frac{\mu_0 i a \sin\theta}{2r^2}$$

$$\therefore B = \frac{\mu_0 i a \times \frac{a}{r}}{2r^2} \quad (1)$$

$\because$ In ΔOPS
$\sin\theta = \frac{a}{r}$

$$B = \frac{\mu_0 i a^2}{2r^3} \quad (2)$$

In ΔOPS;

$$r^2 = a^2 + x^2$$

$$r = (a^2+x^2)^{1/2}$$

OR

$$r^3 = (a^2+x^2)^{3/2}$$

Put the value of r^3 in eq^n. (2)

$$\boxed{B = \frac{\mu_0 i a^2}{2(a^2+x^2)^{3/2}}}$$

Let the no. of turns N in the coil. Then, the magnetic field induction will be

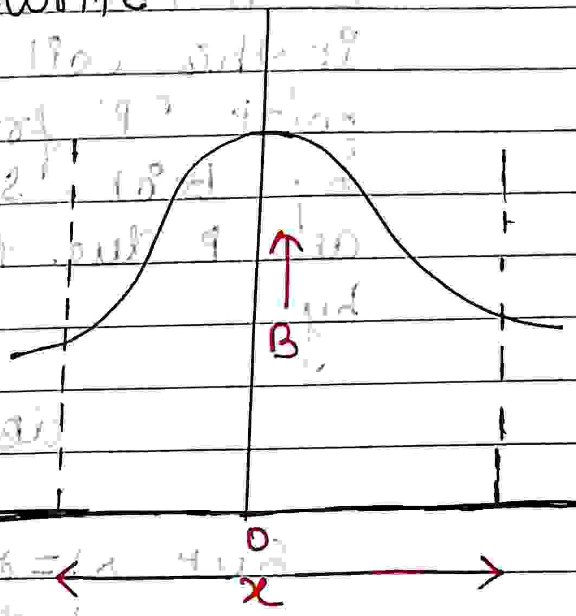
$$\boxed{B = \frac{\mu_0 N i a^2}{2(a^2+x^2)^{3/2}}} \quad \text{—— (3)}$$

It is clear from the above eqn. that the value of B depend on the value of x. Thus if we consider the point 'P' at the centre of coil i.e; $x=0$

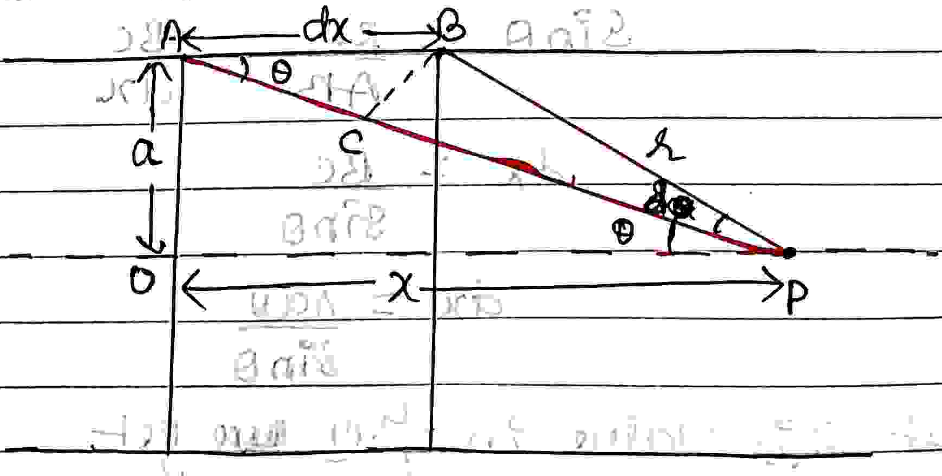
Hence from eqn. (3) we may write

$$\boxed{B = \frac{\mu_0 N i}{2a}}$$

This is the maximum value of magnetic field.



3. To find the magnetic field due to a long current carrying solenoid



let us consider a long solenoid of radius 'a' meter and carrying a current of 'i' Amp. let 'n'

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be the no. of turns per meter length of the solenoid. Let P be a point on the axis of the solenoid at which the field is to be find out.

Let us imagine the solenoid to be divided up into a no. of narrow coils and consider one such coil AB of width dx. The no. of turns in this coil is ndx. Let x be the distance of the point 'P' from the centre 'O' of this coil. Then by Biot-Savart's law, the magnetic induction at P due to this elementary coil is given by

$$dB = \frac{\mu_0 N i a^2}{2(a^2+x^2)^{3/2}}$$

But $N = ndx$

$$\therefore dB = \frac{\mu_0 n i a^2}{2(a^2+x^2)^{3/2}} dx \quad \text{——} \quad (1)$$

In ΔABC

$$\sin\theta = \frac{BC}{AB} = \frac{BC}{dx}$$

$$dx = \frac{BC}{\sin\theta}$$

$$dx = \frac{r\,d\theta}{\sin\theta}$$

[$\because$ $\angle le = \frac{arc}{radius}$

$d\theta = \frac{BC}{r}$

$BC = r\,d\theta$]

Put this value in eqn. (1) we get

$$dB = \frac{\mu_0 n i a^2}{2(a^2+x^2)^{3/2}} \cdot \frac{r\,d\theta}{\sin\theta}$$

[In ΔPAO

$r^2 = (a^2+x^2)$]

$$\therefore dB = \frac{\mu_0 n i a^2}{2(r^2)^{3/2}} \cdot \frac{r\,d\theta}{\sin\theta}$$

$$= \frac{\mu_0 n i a^2}{2r^2} \frac{d\theta}{\sin\theta}$$

$$= \frac{\mu_0 n i}{2} \left[\frac{a}{r}\right]^2 \frac{d\theta}{\sin\theta}$$

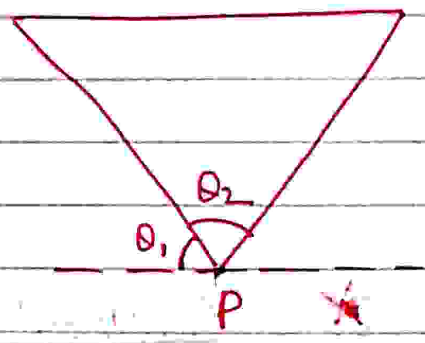
But from figure

$$\frac{a}{r} = \sin\theta$$

$$\therefore dB = \frac{\mu_0 n i}{2} \sin^2\theta \cdot \frac{d\theta}{\sin\theta}$$

$$dB = \frac{1}{2} \mu_0 n i \sin\theta \, d\theta$$

Then the field induction B at P due to the whole solenoid is given by



$$B = \int dB$$

$$= \frac{1}{2} \mu_0 n i \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$

$$= \frac{1}{2} \mu_0 n i \left(-\cos\theta\right)_{\theta_1}^{\theta_2}$$

$$\boxed{B = \frac{1}{2} \mu_0 n i \left(\cos\theta_1 - \cos\theta_2\right)}$$

Case: 1 When the length of solenoid is very long, then

$\theta_1 = 0$, $\theta_2 = \pi$

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$$\therefore B = \frac{1}{2}\mu_0 n i \left(\cos 0 - \cos \pi\right)$$

$$= \frac{1}{2}\mu n i \,(1+1)$$

$$\boxed{B = \mu_0 n i}$$

se: II At the end of the solenoid

$\theta_1 = 0,\ \theta_2 = 90^\circ$

$$B = \frac{1}{2}\mu_0 n i \left(\cos 0 - \cos 90^\circ\right)$$

$$\boxed{B = \frac{1}{2}\mu_0 n i}$$

: III At the first end of the solenoid

$\theta_1 = 90^\circ,\ \theta_2 = 180^\circ$

$$\boxed{B = \frac{1}{2}\mu_0 n i}$$